ment between measured and predicted data for both magnitude and location of the peak. It appears that en route noise level of the single rotation propeller is primarily determined only by the blade passing frequency at high altitude cruise missions.

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Three-Dimensional Laminar Boundary Layer in a Constant Pressure Diverging Flow—Blasius Equivalent

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Introduction

ALTHOUGH a variety of three-dimensional (3D) laminar boundary layer problems have been addressed in the past, we consider here the development of a laminar boundary layer under streamline divergence but constant pressure. This interesting problem has arisen out of an experiment designed[1] to study the effect of streamline divergence, among other parameters associated with general 3D flows, on transitional flow characteristics. The streamline divergence effect under constant pressure is achieved[1,2] by modifying the wind tunnel. As shown in Fig. 1, a distorted section is created with one diverging side wall and a converging roof, thus providing lateral streamline divergence under constant pressure in the region of measurements made on a flat plate. A few typical surface streamlines from flow visualization study[2] are also shown in Fig. 1. Also it was reported that there was no skewing between the surface streamlines and those outside the boundary layer.[2]

The relevance of the present problem, apart from being basic in nature, is seen in the divergent channel flow, albeit with pressure gradient. Kehl,[3] for example, studies turbulent flow in a divergent channel, using a momentum integral method. Kehl[3] considers the flow to emanate from a fictitious source and further considers the flow along a streamline. Along a streamline the spanwise velocity is zero, but its spanwise derivative, which appears in the continuity equation, is nonzero. Thus the boundary layer momentum equation along a streamline is the same as that in 2D flow; however the 3D effect is introduced through the continuity equation.

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Analysis

For the flow under consideration—streamline divergence under constant pressure—we also assume the flow to emanate from a source and consider the flow along a streamline direction (x). As shown in Fig. 2, the source is located at a distance A upstream of the origin of coordinates (x, z); z is the coordinate spanwise to x. The normal direction to the plate is denoted by y. Note that because this is a streamline based coördinate system, it changes from streamline to streamline.

Under the boundary layer approximation the continuity and momentum equations for the flow in the distorted duct with zero pressure gradient are[3]

$$u_x + v_y + u/(A + x) = 0 \tag{1}$$

$$uu_x + vu_y = \nu u_{yy} \tag{2}$$

where u and v denote, respectively, the velocities in x and y directions, ν denotes the kinematic viscosity, and subscripts x and y denote partial derivatives with respect to x and y, respectively. Note that the term $u/(A + x)$ in the continuity Eq. (1) corresponds to the z-derivative of the spanwise velocity component from a kinematical consideration. The boundary conditions for Eq. (2) are

$$\begin{aligned} u = v = 0 \quad &\text{at} \quad y = 0 \\ u \to U \quad &\text{as} \quad y \to \infty \end{aligned} \tag{3}$$

where U denotes the free-stream velocity along a streamline.

To solve Eqs. (1–3), we seek a similarity solution in terms of the similarity variables

$$\eta = y/\delta(x) \qquad u = Uf'(\eta) \tag{4}$$

where $\delta(x)$ is a measure of the local boundary layer thickness, and prime denotes derivative with respect to η.

With v from Eq. (1) and Eq. (4) as

$$v = U(\mathrm{d}\delta/\mathrm{d}x)[\eta f' - f] - [Uf\delta/(A + x)] \tag{5}$$

the momentum Eq. (2) becomes

$$f''' + ff''\{U\delta(\mathrm{d}\delta/\mathrm{d}x)/\nu + U\delta^2/[\nu(A + x)]\} = 0 \tag{6}$$

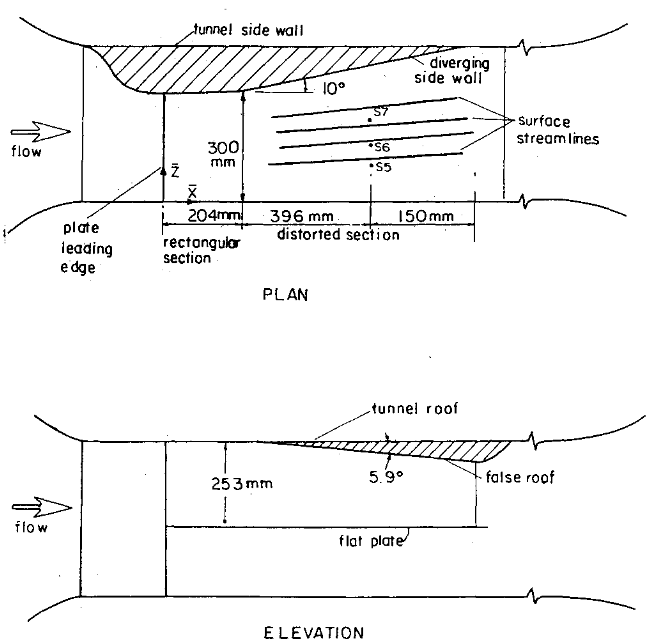

Fig. 1 Schematic view of the distorted test section with a few typical surface streamlines and measuring stations.

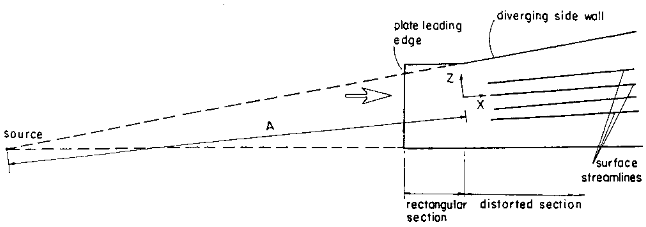

Fig. 2 Details of the streamline based coordinate system and source location.

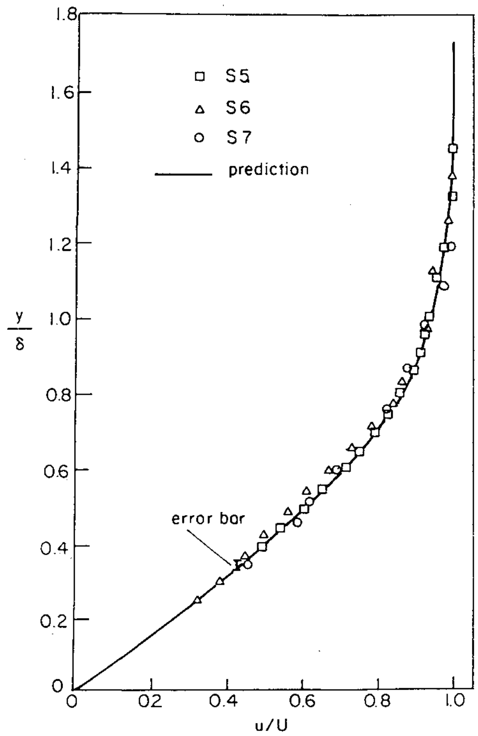

Fig. 3 Comparison of the measured and predicted velocity profiles.

For similarity we require that the coefficient of ff'' in Eq. (6) be a constant (k, say). Solving the resulting differential equation for δ we get

$$\delta^2 = 2k\nu(A + x)/3U + B(A + x)^{-2} \tag{7}$$

where B is a constant of integration that can be evaluated from the value of δ at an initial location.

Upon this, (6) reduces to

$$f''' + kff'' = 0 \tag{8}$$

The boundary conditions are

$$f = f' = 0 \quad \text{at} \quad \eta = 0$$
$$f' \to 1 \quad \text{as} \quad \eta \to \infty \tag{9}$$

Note that as k is a free constant, we could as well set $k = 1$ without any loss of generality leading to the Blasius equation[4]

$$f''' + ff'' = 0 \tag{10}$$

Thus the Blasius equation represents the similarity equation for the 3D flow with streamline divergence under constant pressure.

For the sake of completeness of the present analysis, velocity measurements are made and compared with the theory as discussed in the following section.

Comparison with Experiment

Velocity measurements are made on a flat plate placed in the modified wind tunnel, shown in Fig. 1, available in the Aerospace Engineering Department of the Indian Institute of Science. The freestream turbulence level is less than 0.05 percent. Whereas the total pressure is measured by a pitot probe of diameter(d) = 0.55 mm, the static pressure is measured by a disc probe. A projection manometer capable of reading a pressure difference of 0.1 mm of alcohol is used. For the sake of convenience, the measuring stations are designated by the distance ($\bar{X}$) from the plate leading edge, as shown in Fig. 1.; the corresponding spanwise locations of the measuring stations are denoted by $\bar{Z}$. Although measurements are made at various ($\bar{X}$, $\bar{Z}$) locations, we will present here data for stations designated as $S5(\bar{X} = 600$ mm, $\bar{Z} = 105$ mm), $S6(\bar{X} = 600$ mm, $\bar{Z} = 170$ mm), and $S7(\bar{X} = 600$ mm, $\bar{Z} = 240$ mm), as shown in Fig. 1, for the sake of simplicity; measurements at other stations are available elsewhere.[5] For stations ($S5$) and ($S6$), U = 13 m/s. For station ($S7$), U = 5.5 m/s as the flow at U = 13 m/s had a tendency to become turbulent. The corrections due to displacement effects on the pitot tube due to shear flow and the wall proximity effect are neglected as they are found to be very small: the former is about 0.15 d, and the latter results in a correction of less than 2 percent in total pressure when the axis is at a distance of 0.5 d from the wall. These numbers are obtained from Goldstein.[6] The transverse distance y is normalized with δ corresponding to $0.95U$.

In Fig. 3, the measured velocity profiles in the normalized coordinates at ($S5$), ($S6$), and ($S7$) are compared with the present similarity solution; the bar in this figure indicates the possible range of error. It can be seen that the similarity solution by Eq. (10) is in excellent agreement with the measurements. Note that because the similarity equation does not have explicitly A as a parameter (which is a measure of divergence angle), the normalized velocity profiles for different angles of divergence collapse to a single curve.

Conclusion

A similarity solution is obtained for laminar 3D constant pressure flow with lateral streamline divergence. The similarity solution is shown to reduce to a Blasius solution for 2D flow over a flat plate. Measurements of velocity profiles are made to compare the similarity solution and are found to be in excellent agreement with the prediction.

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